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LAGRANGIAN DESCRIPTION OF N=2 MINIMAL MODELS AS CRITICAL POINTS OF LANDAU-GINZBURG THEORIES

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ABSTRACT

We discuss a two-dimensional lagrangian model with $N=2$ supersymmetry described by a Kähler potential $K(X,\bar{X})$ and superpotential gX^k which explicitly exhibits renormalization group flows to infrared fixed points where the central charge has a value equal that of the $N=2$, A_{k-1} minimal model. We consider the dressing of such models by N=2 supergravity: in contradistinction to bosonic or $N=1$ models, no modification of the β-function takes place.

It has been recognized for some time that $N=2$ minimal models can be viewed as critical points of Landau-Ginzburg theories, and a considerable body of literature has developed around this idea [1,2,3,4,5,6,7]. It is generally believed that in a field-theoretic language such models are described, at and away from the fixed points, by $N=2$ superspace actions of the form

$$\mathcal{S} = \int d^2x d^4\theta \; K(X,\bar{X}) + \int d^2x d^2\theta \; W(X) + \int d^2x d^2\bar{\theta} \; \bar{W}(\bar{X}) \qquad (1)$$

where $K(X,\bar{X})$ is the Kähler potential, function of chiral and antichiral superfields X, $\bar{X}$, while the superpotential $W(X)$ is a quasi-homogeneous polynomial in the chiral superfields. These ideas have been tested in numerous ways, but no complete lagrangian models have been constructed which exhibit explicitly this behavior.

Away from the fixed points, along the renormalization group trajectories, the $N=2$ nonrenormalization theorem ensures that the form of the superpotential remains unchanged while the Kähler potential flows according to quantum corrections in such a way that at the fixed points the resulting action describes superconformally invariant systems. In a complete lagrangian description one would like to exhibit a suitable Kähler potential such that, for example, for a simple Landau-Ginzburg superpotential gX^k the system flows to an infrared fixed point where the central charge is the one of the A_{k-1} N=2 minimal model. We present here such a Kähler potential. Generalizations to other minimal models are straightforward.[8]

We first examine the situation at the fixed point. In the absence of the superpotential W, a generic $N = 2$ σ-model is classically (super)conformally invariant. In the presence of the superpotential the stress-energy tensor (the supercurrent actually) acquires a classical trace (a supertrace) and for a general $K(X, \bar{X})$ no improvement term can be found to make the theory superconformally invariant. In fact, for a given superpotential, the condition of scale invariance fixes the Kähler potential and the improvement term up to a normalization factor. As we shall see, it is this normalization factor that determines the central charge, and it is a specific normalization factor that gets selected, when we start away from the critical point, by the renormalization group flow. (The critical point form of our lagrangian has been also described by Marshakov [5] and, in its Liouville version, by Liao and Mansfield [6], but in these references the normalization factor could not be determined.)

We work in Minkowski space with light-cone variables

$$x^{+\!\!\!+} = \frac{1}{\sqrt{2}}(x^0 + x^1) \quad , \quad \partial_{+\!\!\!+} = \frac{1}{\sqrt{2}}(\partial_0 + \partial_1)$$
$$x^{=} = \frac{1}{\sqrt{2}}(x^0 - x^1) \quad , \quad \partial_{=} = \frac{1}{\sqrt{2}}(\partial_0 - \partial_1) \tag{2}$$

and

$$\Box \equiv \partial^\mu \partial_\mu = 2\partial_{+\!\!\!+}\partial_{=} \quad , \quad \partial_{=}\frac{1}{x^{+\!\!\!+}} = 2\pi i \delta^{(2)}(x) \tag{3}$$

The superspace spinorial coordinates are θ^+, θ^-, $\bar{\theta}^+$, $\bar{\theta}^-$, and the corresponding covariant derivatives satisfy

$$\{D_+, \bar{D}_+\} = i\partial_{+\!\!\!+} \quad , \quad \{D_-, \bar{D}_-\} = i\partial_{=} \tag{4}$$

with all other anticommutators vanishing. For a kinetic term $\int d^2x d^4\theta \; X\bar{X}$ the chiral field propagator is

$$< X(x,\theta)\bar{X}(0) >= -\frac{1}{2\pi}\bar{D}^2 D^2 \delta^{(4)}(\theta) \ln[m^2(2x^{+\!\!\!+}x^{=} + \ell^2)] \tag{5}$$

where m and ℓ are infrared and ultraviolet cutoffs respectively. We have defined $D^2 \equiv D_+D_-$ and $\bar{D}^2 \equiv \bar{D}_+\bar{D}_-$.

We couple the system described by Eq. (1) to $N = 2$ supergravity and in order to conveniently describe the above-mentioned improvement of the supercurrent we include a chiral "dilaton" term, so that the action takes the form

$$\begin{aligned} \mathcal{S} &= \int d^2x d^4\theta E^{-1} K(e^{iH.\partial}X, e^{-iH.\partial}\bar{X}) + \int d^2x d^2\theta e^{-2\sigma} W(X) \\ &+ \int d^2x d^2\bar{\theta} e^{-2\bar{\sigma}} \bar{W}(\bar{X}) + \int d^2x d^2\theta \; e^{-2\sigma} R \; \Psi(X) + \int d^2x d^2\bar{\theta} \; e^{-2\bar{\sigma}} \bar{R} \; \bar{\Psi}(\bar{X}) \end{aligned} \tag{6}$$

Here the vector superfield H and the chiral compensator σ are the supergravity prepotentials. At the linearized level we have the explicit expressions [8]

$$\begin{aligned} E^{-1} &= 1 - [\bar{D}_+, D_+]H_{=} - [\bar{D}_-, D_-]H_{+\!\!\!+} \\ R &= 4\bar{D}_+\bar{D}_-[\bar{\sigma} + D_+\bar{D}_+H_{=} + D_-\bar{D}_-H_{+\!\!\!+}] \\ \bar{R} &= 4D_+D_-[\sigma - \bar{D}_+D_+H_{=} - \bar{D}_-D_-H_{+\!\!\!+}] \end{aligned} \tag{7}$$

The general solution of the constraints of $N = 2$ supergravity is given in Ref. 9.

The invariance of the supergravity-coupled system under local supersymmetry transformations is expressed by the conservation law

$$\bar{D}_- J_{\ddagger} = D_+ J \quad , \quad D_- J_{\ddagger} = \bar{D}_+ \bar{J} \tag{8}$$

where the supercurrent is given by

$$J_{\ddagger} \equiv \frac{\delta \mathcal{S}}{\delta H_{=}}|_{H,\sigma=0} = 2[D_+ X \bar{D}_+ \bar{X} K_{X\bar{X}} - 2\bar{D}_+ D_+ \Psi + 2D_+ \bar{D}_+ \bar{\Psi}] \tag{9}$$

and the supertrace by

$$J \equiv \frac{\delta \mathcal{S}}{\delta \sigma}|_{H,\sigma=0} = -2[W - 2\bar{D}_+ \bar{D}_- \bar{\Psi}] \tag{10}$$

We have introduced the Kähler metric

$$K_{X\bar{X}} = \frac{\partial^2 K}{\partial X \partial \bar{X}} \tag{11}$$

Superconformal invariance requires the supertrace J to vanish. For the superpotential $W = gX^k$ the equations of motion (with the notation $K_X \equiv \partial_X K$, etc.)

$$\bar{D}_+ \bar{D}_- K_X + W_X = 0 \tag{12}$$

give

$$W = -\frac{1}{k} \bar{D}_+ \bar{D}_- (X K_X) \tag{13}$$

Using this expression in Eq. (10), the condition for superconformal invariance, $J = 0$, $\bar{J} = 0$, requires

$$X K_X = -2k \bar{\Psi}(\bar{X}) \quad , \quad \bar{X} K_{\bar{X}} = -2k \Psi(X) \tag{14}$$

modulo a *linear* superfield which gives no contributions to the action. We have assumed that Ψ, $\bar{\Psi}$ are local, and (anti)chirality and dimensionality require them to be functions of X, $\bar{X}$ respectively. The equations above can be immediately integrated and give

$$\begin{aligned} K &= \alpha \ln X \ln \bar{X} \\ \Psi &= -\frac{\alpha}{2k} \ln X \quad , \quad \bar{\Psi} = -\frac{\alpha}{2k} \ln \bar{X} \end{aligned} \tag{15}$$

with arbitrary constant α. Using the field redefinition $X \equiv e^{\Phi}$ the corresponding lagrangian can be recast in Liouville form.

We compute now the conformal anomaly of our model, and show that the central charge of the fixed point theory equals the central charge of $N = 2$, A_{k-1} minimal models when α is suitably chosen. It is given by the coefficient in front of the induced supergravity effective action $R \Box^{-1} \bar{R}$ obtained by integrating out the fields X, $\bar{X}$, and

can be determined by contributions to the $H_=$ self-energy, from which the covariant expression can be reconstructed.

We compute away from the fixed point, using an effective configuration-space propagator

$$< X(x,\theta)\bar{X}(x',\theta') >= -\frac{K^{X\bar{X}}}{2\pi}\bar{D}^2 D^2 \delta^{(4)}(\theta-\theta')\ln\{m^2[2(x-x')^{\ddagger}(x-x')^{=}+\ell^2]\} \quad (16)$$

where $K^{X\bar{X}}$ is the inverse of the Kähler metric (cf. Ref. 12, Eq. (3.13); additional terms, involving derivatives of the Kähler metric in the propagator do not give relevant contributions). The couplings to $H_=$ can be read from the supercurrent in Eq. (9). The relevant vertex from the Kähler potential is

$$2i\int d^4\theta \; H_= D_+ X \bar{D}_+ \bar{X} K_{X\bar{X}} \quad (17)$$

This leads to the one-loop contribution

$$-\frac{1}{\pi^2}H_= \frac{\bar{D}_- D_- \bar{D}_+ D_+}{(x-x')^2_{\ddagger}} H_= \quad (18)$$

From the dilaton term coupling

$$4\int d^4\theta(\bar{\Psi}-\Psi)\partial_{\ddagger} H_= \quad (19)$$

we have the tree level contribution

$$-\frac{16}{\pi}\Psi_X K^{X\bar{X}}\bar{\Psi}_{\bar{X}} H_= \frac{\bar{D}_- D_- \bar{D}_+ D_+}{(x-x')^2_{\ddagger}} H_= \quad (20)$$

Using the relation between the Kähler potential and the improvement term at the fixed point given in Eq. (15), we obtain the final result

$$-\frac{1}{\pi^2}H_= \frac{\bar{D}_- D_- \bar{D}_+ D_+}{(x-x')^2_{\ddagger}} H_= \left(1+\frac{4\pi\alpha}{k^2}\right) \Rightarrow \frac{1}{4\pi}R\frac{1}{\Box}\bar{R}\left(1+\frac{4\pi\alpha}{k^2}\right) \quad (21)$$

From this expression we obtain the central charge of the system

$$c = 1+\frac{4\pi\alpha}{k^2} \quad (22)$$

For $\alpha = -\frac{k}{2\pi}$ it equals the correct value for the $N=2$, A_{k-1} minimal model,

$$c = 1-\frac{2}{k} \quad (23)$$

We exhibit now a system which flows in the IR region to the superconformal theory defined above. It has two properties: its β-function is one-loop exact, so

that we can make all-orders statements about the flows; and, although it contains arbitrary parameters, the flow to the fixed point uniquely picks out values which give the correct central charge for identification with the A_{k-1} minimal models. The theory is described by the superpotential gX^k and the Kähler metric

$$K_{X\bar{X}} = \frac{1}{1 + bX\bar{X} + c(X\bar{X})^2} \tag{24}$$

corresponding to the Kähler potential

$$K = \int dX d\bar{X} K_{X\bar{X}} = X\bar{X} - \frac{b}{4}(X\bar{X})^2 + \frac{b^2 - c}{9}(X\bar{X})^3 + \cdots \tag{25}$$

Quantum corrections give rise to divergences that can be reabsorbed by renormalization of the parameters b, c, and wave-function renormalization. Actually it is convenient to rescale the field, $X \to a^{-\frac{1}{2}}X$, so that the Kähler metric and superpotential become (with a redefinition of the parameter c)

$$K_{X\bar{X}} = \frac{1}{a + bX\bar{X} + c(X\bar{X})^2} \qquad , \qquad ga^{-\frac{k}{2}}X^k \tag{26}$$

(A related metric, with $a = c$, has been discussed in a bosonic σ-model context by Fateev *et al* [10]. The authors of Ref. 11 have speculated on the relevance of such metrics for studying $N = 2$ flows.).

Thus as in a standard σ-model approach one renormalizes the Kähler metric including the parameter a (this is equivalent to wave-function renormalization). According to Eq. (26), since the superpotential is not renormalized, a renormalization of the parameter a leads to a corresponding renormalization of the coupling constant g.

At the one-loop level the divergence is proportional to the Ricci tensor,

$$-(\frac{1}{2\pi}\ln m^2\ell^2)R_{X\bar{X}} = (\frac{1}{2\pi}\ln m^2\ell^2)\frac{ab + 4acX\bar{X} + bc(X\bar{X})^2}{[a + bX\bar{X} + c(X\bar{X})^2]^2} \tag{27}$$

so that the Kähler metric, including one-loop corrections, becomes

$$K_{X\bar{X}} + \Delta K_{X\bar{X}} = \frac{1}{a(1-\Lambda b) + (b - 4\Lambda ac)X\bar{X} + c(1-\Lambda a)(X\bar{X})^2} \tag{28}$$

where $\Lambda \equiv \frac{1}{2\pi}\ln m^2\ell^2$. The original parameters in the classical lagrangian are then expressed in terms of renormalized ones:

$$\begin{aligned} a &= Z_a a_R \quad , \quad b = Z_b b_R \quad , \quad c = Z_c c_R \\ g &= \mu Z_g g_R \end{aligned} \tag{29}$$

where μ is the renormalization mass scale, and $Z_g Z_a^{-\frac{k}{2}} = 1$ as required by the $N = 2$ nonrenormalization theorem. From Eq.(28) we find

$$\begin{aligned} Z_a &= 1 + b(\frac{1}{2\pi}\ln\mu^2\ell^2) \\ Z_b &= 1 + \frac{4ac}{b}(\frac{1}{2\pi}\ln\mu^2\ell^2) \\ Z_c &= 1 + b(\frac{1}{2\pi}\ln\mu^2\ell^2) \\ Z_g &= 1 + \frac{bk}{2}(\frac{1}{2\pi}\ln\mu^2\ell^2) \end{aligned} \tag{30}$$

Defining $t = \ln\mu$, the renormalized parameters satisfy the following renormalization group equations (in the following we drop the subscript R)

$$\begin{aligned} \frac{da}{dt} &= -\frac{1}{\pi}ab \\ \frac{db}{dt} &= -\frac{4}{\pi}ac \\ \frac{dc}{dt} &= -\frac{1}{\pi}cb \\ \frac{dg}{dt} &= -(1 + \frac{b}{2\pi}k)g \end{aligned} \tag{31}$$

Conformal invariance is achieved at the zeroes of the coupling β-functions. In particular we are looking for a nontrivial IR fixed point for the coupling constant g, i.e. such that $b(t) \to -\frac{2\pi}{k}$ as $t \to -\infty$. Thus we study the solutions of the system in Eq. (31) and select the relevant trajectories. The equations in (31) have two invariants, the ratio

$$\frac{a}{c} = \rho \tag{32}$$

and the combination, which we choose to make positive and parametrize suitably,

$$b^2 - 4ac = b^2 - 4\rho c^2 = (\pi\lambda)^2 \tag{33}$$

Here ρ and λ are arbitrary constants parametrizing individual trajectories.

In the b-c plane we obtain two types of trajectories, hyperbolas or ellipses, depending on the sign of ρ. Since we are interested in trajectories with two fixed points we write the elliptical solutions, with $\rho < 0$. (The bosonic model studied in Ref. 10, written in a different coordinate system, has $\rho = 1$.)

$$\begin{aligned} b(t) &= \pi\lambda\tanh\lambda t \\ a(t) &= \pm\frac{\pi\lambda\sqrt{-\rho}}{2}(\cosh\lambda t)^{-1} \\ c(t) &= \mp\frac{\pi\lambda}{2\sqrt{-\rho}}(\cosh\lambda t)^{-1} \\ g(t) &= g_0 e^{-t}[\cosh\lambda t]^{-\frac{k}{2}} \end{aligned} \tag{34}$$

The wanted IR fixed point is reached by flowing along trajectories which have

$$\lambda = \frac{2}{k} \tag{35}$$

In this case the superfield $a^{-\frac{1}{2}}X$ acquires anomalous dimension $1/k$ in the corresponding IR conformal theory, while a and c flow to zero. Therefore, the effective lagrangian with Kähler potential $K(X, \bar{X}, a(t), b(t), c(t))$ and superpotential $W(X, g(t), a(t))$ has the following behaviour in the infrared,

$$t \to -\infty \quad , \quad K(t) \to -\frac{k}{2\pi} \ln X \ln \bar{X} \quad , \quad W(t) \to g_0 X^k \tag{36}$$

The improvement term at the IR fixed point has $\Psi = \frac{1}{4\pi} \ln X$.

Changing variables, $X \equiv e^{\Phi}$, leads to the Liouville lagrangian

$$\mathcal{L} = -\frac{k}{2\pi} \bar{\Phi}\Phi + g_0 e^{k\Phi} \tag{37}$$

with negative kinetic term and *with normalization determined by the superpotential* (cf. Refs. 5,6).

We emphasize that imposing conformal invariance at the one-loop level, i.e. $R_{X\bar{X}} = 0$, is sufficient to insure the absence of divergences at higher-loop orders since the Riemann tensor trivially vanishes as well. Moreover, while in the bosonic or in the $N = 1$ supersymmetric theories the dilaton term contributes to the metric β-function, in the $N = 2$ case no metric-dilaton mixing occurs due to the chirality of Ψ. Thus at the conformal point we obtain exact, all-order results.

The case of two fields, with superpotential $gX^n + g'X^kY^m$ can be treated in similar fashion [8]. The flows to the IR fixed point lead to a central charge which agrees with the results for the various minimal models described by a two-field Landau-Ginzburg potential.

At the fixed point the model we have discussed is conformally invariant and therefore integrable. We have examined its integrability along the flow, by looking for a higher-spin conserved current. We observe that for the case $k = 1$ the model reduces to the supersymmetric complex sine-Gordon system studied by Napolitano and Sciuto [13], which is indeed *classically* integrable. In particular, these authors have shown that a spin 3/2 conserved current exists for their model. However, for $k > 1$ we have been unable to construct a conserved spin 3/2 current. Although we suspect that no conserved current exists for our models, integrability along the flows remains an open question.

Finally, we discuss effects due to gravitational dressing [14,15,16,17]. It has been shown that for bosonic theories one-loop β-functions in the presence of induced gravity are related to the corresponding ones computed in the absence of gravitational effects by the universal formula

$$\beta_G = \frac{\kappa + 2}{\kappa + 1} \beta \tag{38}$$

Here κ is the level of the gravitational $SL(2,R)$ current algebra which can be expressed in terms of the matter central charge as

$$\kappa + 2 = \frac{1}{12}[c - 13 - \sqrt{(1-c)(25-c)}] \tag{39}$$

In the semiclassical limit, $c \to -\infty$, the dressing becomes

$$\beta_G \to (1 + \frac{6}{c})\beta \tag{40}$$

The above result was obtained by treating induced gravity in light-cone gauge and making use of the corresponding Ward identities [15], or in conformal gauge [16] where induced gravity is described by the Liouville action, making use of the fact that one must distinguish between the scale defined by the fiducial metric, and the physical scale defined by the Liouville field. It can also be checked explicitly in perturbation theory by performing a calculation for the standard bosonic σ-model coupled to induced gravity in light-cone gauge.

A similar treatment is possible for $N = 1$ and $N = 2$ induced supergravity [18]. The argument is particularly simple in (super)conformal gauge. The idea [16,17] is that, in the presence of the Liouville mode, the physical scale gets modified with respect to the standard renormalization scale. In two-dimensional gravity the only dimensionful object, which provides the physical scale, is the cosmological constant term. In light-cone gauge the cosmological constant is just a c-number, so that the usual scaling is physical and the modifications of the matter β-functions arise through new divergent contributions due to the gravitational couplings. In conformal gauge instead the one-loop matter divergence does not receive gravitational corrections. However in this case the cosmological term is renormalized and thus it determines the new physical scale that enters in the computation of the dressed matter β-functions.

In (super)conformal gauge the computation can be performed treating on equal footing the bosonic, $N = 0$, and $N = 1$, 2 induced (super)gravities. In all cases the presence of the Liouville field ϕ in the cosmological constant term determines the physical scale Λ through

$$\int d^2z d^{2N}\theta e^{\alpha_+\phi} \Leftrightarrow \int d^2z d^{2N}\theta \Lambda^{-2s} \tag{41}$$

where α_+ is the positive solution of

$$-\frac{1}{2}\alpha(\alpha + Q) = s \tag{42}$$

i.e.

$$\alpha_+ = \frac{1}{2}\left[-Q + \sqrt{Q^2 - 8s}\right] \tag{43}$$

For the various theories of interest one has, from the dimensionality of $d^2zd^{2N}\theta$, $s = \frac{1}{2}(2 - N)$. Thus,

$$N = 0 : \quad s = 1 \qquad Q = \sqrt{\frac{25-c}{3}}$$

$$
\begin{aligned}
N &= 1: \quad s = \frac{1}{2} \qquad Q = \sqrt{\frac{9-c}{2}} \\
N &= 2: \quad s = 0 \qquad Q = \sqrt{\frac{1-c}{2}}
\end{aligned}
\tag{44}
$$

The (super)gravity modification of one-loop matter β-functions comes from the chain rule relating derivatives with respect to the physical scale Λ and the renormalization scale μ in the absence of gravity

$$
\beta_G = \frac{\partial ln\mu}{\partial ln\Lambda}\beta = -\frac{2s}{\alpha_+ Q}\beta
\tag{45}
$$

Using the above expressions, one finds for the ordinary gravity case, $N = 0$, the result in Eq. (38). For $N = 1,\ 2$ using also the expressions for the level κ of the light-cone supergravity Kač-Moody algebra

$$
\begin{aligned}
N = 1: \qquad \kappa + \frac{3}{2} &= \frac{1}{8}\left[c - 5 - \sqrt{1-c)(9-c)}\right] \\
N = 2: \qquad \kappa + 1 &= \frac{1}{4}(c-1)
\end{aligned}
\tag{46}
$$

one finds

$$
\begin{aligned}
N = 1: \qquad \beta_G &= \frac{\kappa + \frac{3}{2}}{\kappa + 1}\beta \\
N = 2: \qquad \beta_G &= \beta
\end{aligned}
\tag{47}
$$

These results can be verified perturbatively in light-cone gauge. Details are presented in a separate publication [18].

We conclude that there is no supergravity dressing of β-functions in the $N = 2$ case. We also note that even in the presence of supergravity the nonrenormalization theorems continue to hold so that there is no correction to the superpotential. Therefore, the RG flow results discussed above are not modified by the presence of induced $N = 2$ supergravity.

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